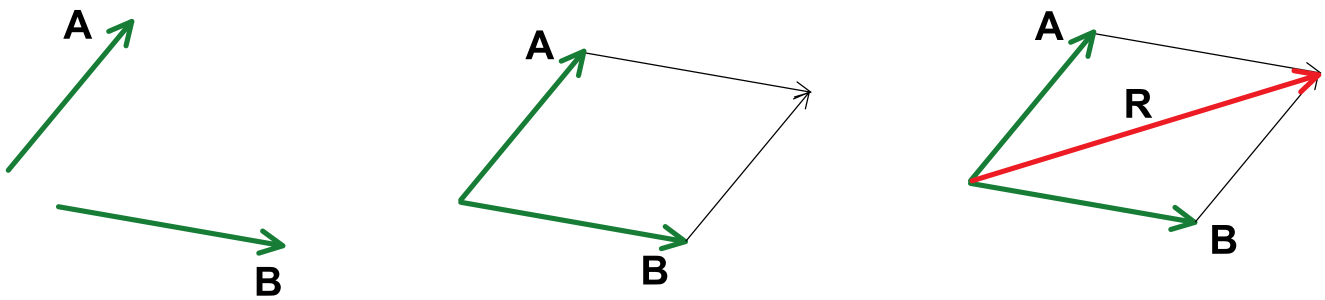
Vector Operations (See Part A)

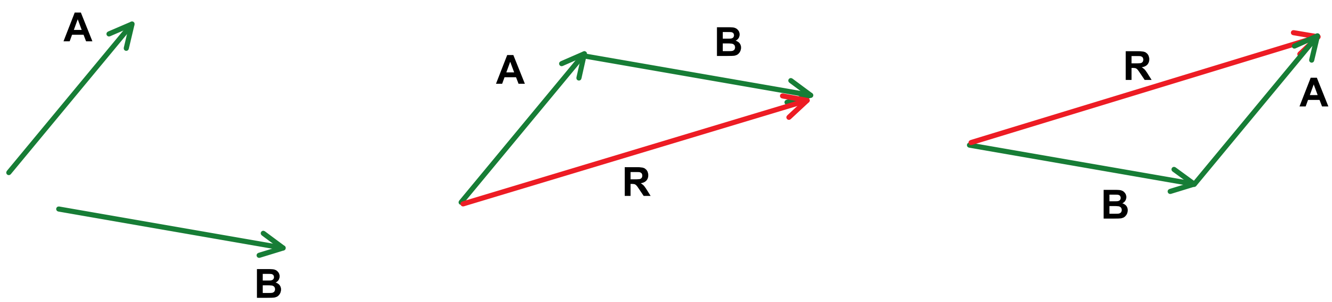
Addition:

- Use parallelogram law or triangle rule
  - Parallelogram law: Tail to tail



R = A + B

  - Triangle rule: Head to tail

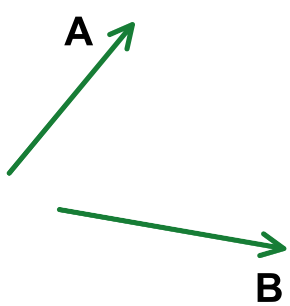


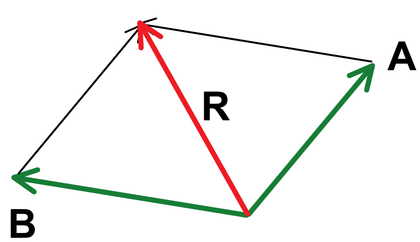
R = A + B

R = B + A

Subtraction:

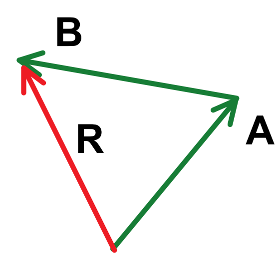
- Use parallelogram law or triangle rule





R = A - B

Parallelogram law



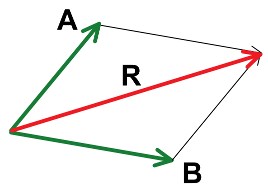
R = A - B

Triangle rule

Resultant Force (See Part A)

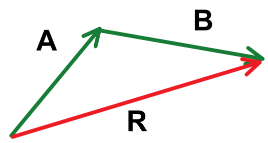
i) Graphical Method:

- Use the parallelogram law or triangle rule (see part B, ii):
  - Important equations to remember: Law of Sine and Cosine



R = A + B

Parallelogram law



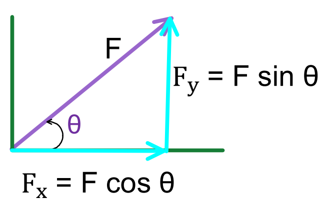
R = A + B

Triangle rule

☺ Note: Use this method to resolve the force or vector into two components. For resultant force, use the next method because it is faster.

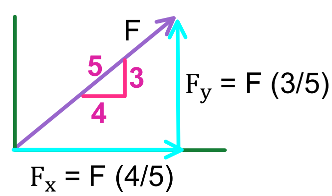
ii) Algebraic Method:

- Use Algebraic sum of the x and y axis: $\mathbf{F_R} = \sqrt{\mathbf{F_x}^2 + \mathbf{F_y}^2}, \boldsymbol{\theta} = \tan^{-1}\left(\frac{\mathbf{F_y}}{\mathbf{F_x}}\right)$
  - Important equations to remember: Trigonometry Functions



Sin θ = $\frac{\text{Opp}}{\text{Hyp}}$, **Cos θ =** $\frac{\text{Adj}}{\text{Hyp}}$, **Tan θ =** $\frac{\text{Opp}}{\text{Adj}}$

θ = $\tan^{-1} = \frac{\text{Opp}}{\text{Adj}} = \frac{F_y}{F_x}$



Similar Triangles:

$\frac{F_x}{4} = \frac{F}{5} = \frac{F_y}{3}$

θ = $\tan^{-1} = \frac{\text{Opp}}{\text{Adj}} = \frac{F_y}{F_x}$

Equations on page 72, Reference Handbook

Cables and Pulleys (See Part B)

A. Steps For Analysis:

i) Draw the Free Body Diagram (**FBD**)

ii) Identify all **forces**

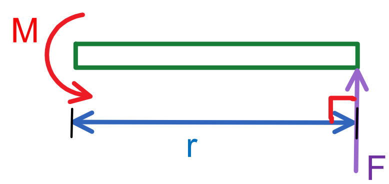
iii) Equilibrium equations: $\sum F_x = 0, \sum F_y = 0.$

Moment of Force (See Part C)

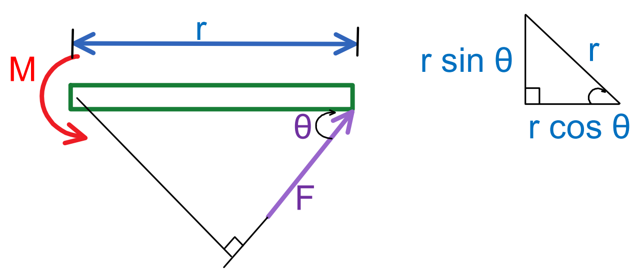
M = r F

- **F** is the **force** and **r** is the **moment arm**, which is the **perpendicular** distance to the **line of action** of the **force**.

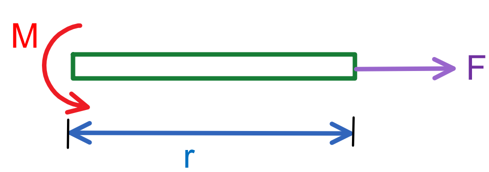
Examples:



M = r F



M = (r sin θ) F

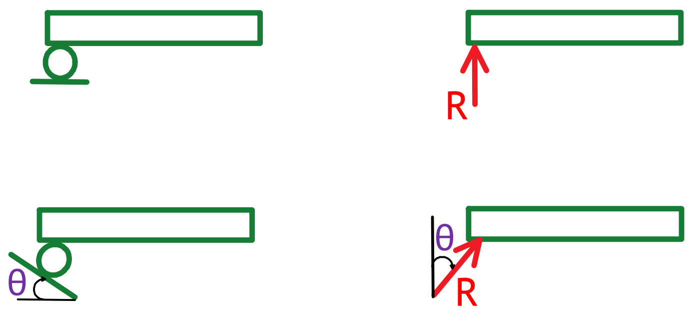


M = r F = (0) F = 0
Same line of action!

Types of Supports (See Part C)

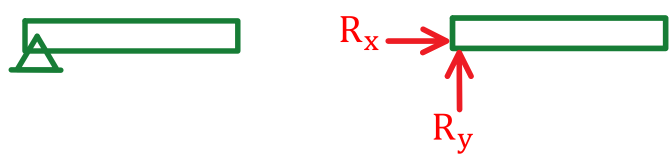
i. Roller:

- ◇ Reaction prevents **translation** in the **vertical** direction.
- ◇ Reaction force acts **perpendicular** to the surface.



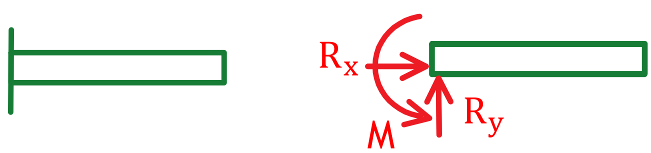
ii. Pin or hinge:

- ◇ Reaction prevents **translation** in **any direction**.
- ◇ The resultant reaction is broken into **y** and **x** components.



iii. Fixed:

- ◇ Reaction prevents **translation** and **rotation**.
- ◇ Reaction has a force (in the **x** and **y** direction) and a **moment**.



Trusses (See Part D)

(I) Identifying Zero Force Members

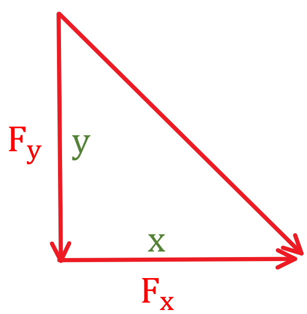
i) If **two noncollinear** members are connected to a joint that is not subjected to **any external loads** or reactions, then **both members** are **zero**.

ii) If **three** members, **two** of which are **collinear** and are connected to a joint that is not subjected to **any external loads** or reactions, then the force **member** that is **not collinear** is **zero**.

(II) Fast Method Of Joint (FMJ)

Four Rules:

1. Equilibrium Equations (Mentally)
2. Break diagonal vectors into 2 components
3. Draw vectors head-head or tail-tail
4. Every action, there is an equal and opposite reaction



$$F_y = \frac{y}{x} F_x$$

$$F_x = \frac{x}{y} F_y$$

(III) Forces on members:

- ◇ If member **elongates** ⟶ **Tension** (Force pulling the joint)
- ◇ If member **shrinks** ⟶ **Compression** (Force pushing into the joint)

(IV) Method of Section

► **Cut** the truss where we want to determine the forces.

► Draw **F**ree **B**ody **D**iagram.

► Take **moment** where two unknown forces **intersect**, so we have only **one unknown** in the moment equation.

Centroids (See Part E)

Procedure For Analysis: Given A Function

1. **Draw** a small **vertical** or a **horizontal** rectangle
2. Find the area of the small rectangle, **A = base x height**
3. Find $\bar{x}$ and $\bar{y}$
4. Plug in to the equation: $\mathbf{x_c} = \frac{\int \bar{x}\, \mathbf{dA}}{\mathbf{A}}$, $\mathbf{y_c} = \frac{\int \bar{y}\, \mathbf{dA}}{\mathbf{A}}$

Procedure For Analysis: Given Geometric Shapes

1. **Break** the drawing into **geometric parts**.
2. Find the centroid, $\mathbf{x_n}$ or $\mathbf{y_n}$ for **each geomteric** part & always with **respect to the datum (0,0)!**
3. Find the **area** of the geometric **shapes**.
4. Plug in to the equations: $\mathbf{x_c} = \frac{\sum \mathbf{x_n}\, \mathbf{a_n}}{\mathbf{A}}$, $\mathbf{y_c} = \frac{\sum \mathbf{y_n}\, \mathbf{a_n}}{\mathbf{A}}$

Moment of Inertia (See Part F)

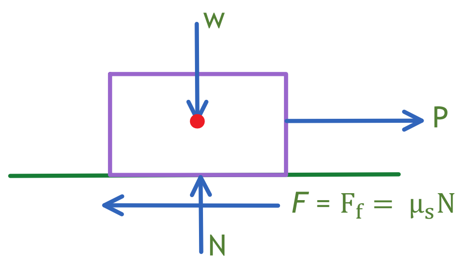
Procedure For Analysis:

1. **Break** the drawing into **geometric parts**.
2. Find the **moment of inertia** for **each** geometric **shape**.
3. Find $\mathbf{d_x}$ and $\mathbf{d_y}$, if necessary!
4. Plug in to the equations: $\mathbf{I'_x} = \mathbf{I_{xc}} + \mathbf{d^2{}_y A}$, $\mathbf{I'_y} = \mathbf{I_{yc}} + \mathbf{d^2{}_x A}$

Centroid about x & y axis	$\mathbf{I_x}$ **or** $\mathbf{I_y}$	$\mathbf{I_{xc}}$ **or** $\mathbf{I_{yc}}$	$\mathbf{d_x}$ **or** $\mathbf{d_y}$
Shape on x or y axis	Use These Equations	Not Applicable	Zero – shape is on the axis, there is NO distance!
Shape NOT on x or y axis	Not Applicable	Use These Equations	Distance from x or y axis to the centroid of the small shape.
Centroid about x & y centroidal axis	$\mathbf{I_x}$ **or** $\mathbf{I_y}$	$\mathbf{I_{xc}}$ **or** $\mathbf{I_{yc}}$	$\mathbf{d_x}$ **or** $\mathbf{d_y}$
Shape is symmetric	Not Applicable	Use These Equations	Zero – centroid of the small shape is at the same point as the centroid of the whole shape!
Shape is NOT symmetric (find centroid x_c and y_c)	Not Applicable	Use These Equations	Distance from centroid of the small shape to the centroid of the whole shape! (Also, use fast method, e.g. $\mathbf{d_x} = (\mathbf{x_c} - \mathbf{x_n})$, see examples in the course)

Friction Force (See Part G)

Free body diagram:



- **Surface** or floor exerts a **normal force** and a **frictional force**.
- **Frictional Force** always **tangent** to the surface.
- For equilibrium, **normal force** acts **upward** and **perpendicular** to the surface to **balance out** the **weight**.
- **Frictional force** acts **opposite** of the **force P** to reach equilibrium and to **prevent motion** to the right.

Friction Forces:

- **F**, the **static frictional force**, is when equilibrium is reached.
- **F**, the **limiting static frictional force**, is when the maximum value is reached but still maintaining equilibrium.
- $\mathbf{F_k}$, the **kinetic frictional force**, is when an object starts moving and sliding occurs

Procedure For Analysis:

i) **Draw** the Free Body Diagram (FBD)

ii) **Identify** all forces

iii) **Equilibrium** and **friction force** equations: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$, $F = \mu_s N$

- Note: **Normal force** is **NOT** Always equal to **weight**.

Belt Friction (See Part G)

- $\mathbf{F_1 = F_2\, e^{\mu\theta}}$, where $\mathbf{F_1 > F_2}$